This question paper contains 4 printed pages.

B.C.A. (Pt. II)

232

Disc. Math.

B.C.A. (PART II) EXAMINATION - 2018

(FACULTY OF SCIENCE)

(Three - Year Scheme of 10+2+3 Pattern)

Paper 232

DISCRETE MATHEMATICS

Time allowed : Three Hours

Maximum Marks : 100

Part I : (Very short answer) consists of 10 questions of 2 marks each. Maximum limit for each question is up to 40 words.

Part II: (Short answer) consists of 5 questions of 4 marks each. Maximum limit for each question is up to 80 words.

Part III : (Long answer) consists of 5 questions of 12 marks each with internal choice.

PART - I

1. Very Short Answer Type Questions

   (a) Convert the decimal number $(156)_{10}$ into binary form.

   (b) Compute the sum $(11011)_2 + (10011)_2$ into decimal form.

   (c) Define union of two sets.

   (d) Define equivalence relation.

   (e) By using truth table, for two statements p and q in usual notations show that $p \vee (p \wedge q) \equiv p$

   (f) For all elements 'a' of Boolean Algebra show that $a + 1 = 1$.

   (g) Define degree of a vertex in graphs.

   (h) What do you mean by proper colouring and chromatic number of a graph.

   (i) Define rooted and binary trees.

   (j) Define minimal spanning tree.

PART - II

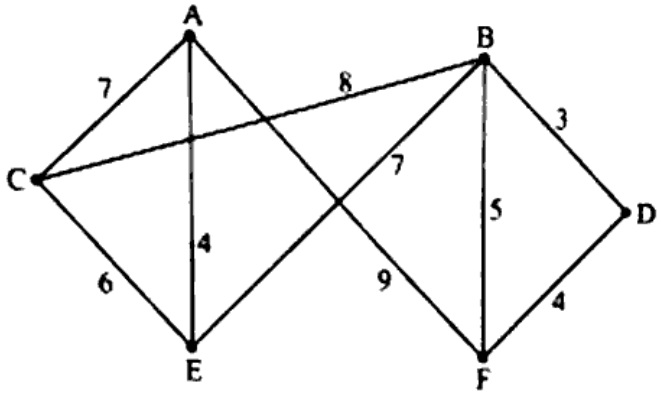
2. Find the coefficient of x^r in the expression $\frac{1}{(x-3)(x-2)^2}$

3. If A and B are any two sets, then prove that : $(A \cup B)' = A' \cap B'$

4. For any two statements p and q show that $(p \wedge q) \Rightarrow (p \vee q)$ is a tautology.

5. If in a graph $G = (V, E)$ there are n vertices and e edges then prove that in the complementary graph $\bar{G}$ the number of vertices will be $\frac{n(n-1)}{2} - e$.

6. Find the minimal spanning tree by Krushal's algorithm in the following graph:



PART - III

7. (a) Compute $(38)_{10} + (69)_{10} = (\ )_2$

(b) Compute $(11011)_2 - (10011)_2 = (\ )_{10}$

(c) Use mathematical induction to prove that the sum of the first n odd positive integers is n^2.

(d) Using generating function find the solution of the recurrence relation.

$a_r - 5a_{r-1} + 6a_{r-2} = 0, r \geq 2, a_0 = 6, a_1 = 30.$

OR

(a) Compute $(11001)_2 + (11101)_2 = (\ )_2$

(b) Compute $(46)_{10} - (146)_{10} = (\ )_2$

(c) Using mathematical induction method prove that :

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}, (n \geq 1)$$

(d) Find the solution of the recurrence relation :

$a_r = 3a_{r-1} + 2^r, r \geq 1, a_0 = 1.$

8. (a) If A, B, C and D are anyfour sets, then prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

(b) If $f : Q \to Q$, $f(x) = 2x$ and $g : Q \to Q$, $g(x) = x + 2$ then verify $(gof)^{-1} = f^{-1} og^{-1}$.

OR

(c) For any three sets A, B and C, show that $A - (B \cup C) = (A - B) \cap (A - C)$.

(f) On the set of real numbers, a binary operation $*$ is defined as

$a * b = a + b + ab$, show that this binary operation is commutative and associative.

9. (a) If p and q are two statements, then by preparing truth table show that the compound statements $p \Leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

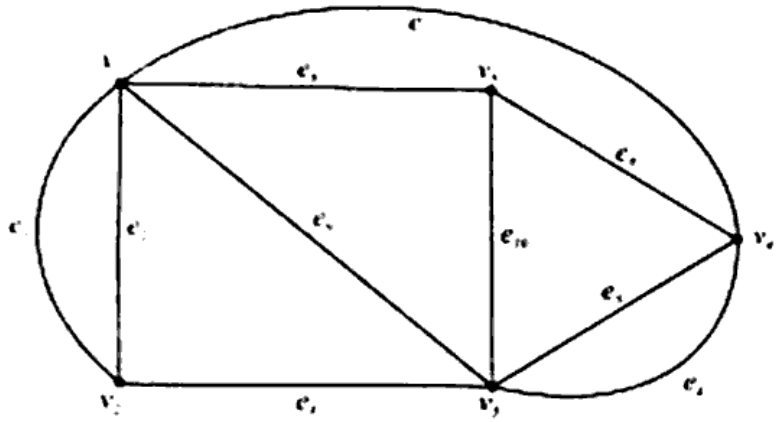
(b) What are the different methods of proving theorems. Prove that $\sqrt{2}$ is irrational number by giving a proof by contradiction.

OR

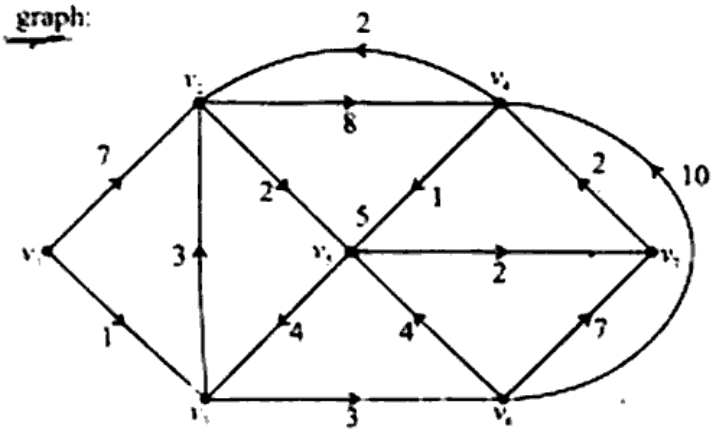
(a) Prove that no Boolean algebra can have exactly three distinct elments.

(b) If a, b, c are any three arbitrary elements of the Boolean algebra $(B, +, ., ')$ such that $a + b = a + c$ and $a.b = a.c$ then prove that $b = c$.

10. (a) Find the incident matrix and adjacency matrix of the following graph:

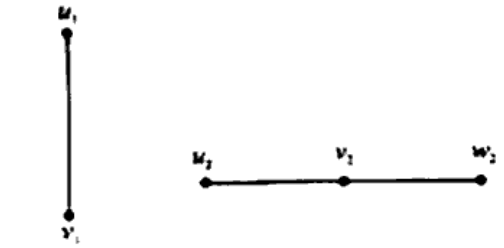


(b) Find the shortest path and shortest distance from the vertices v_1 to v_7 in the following weighted graph:



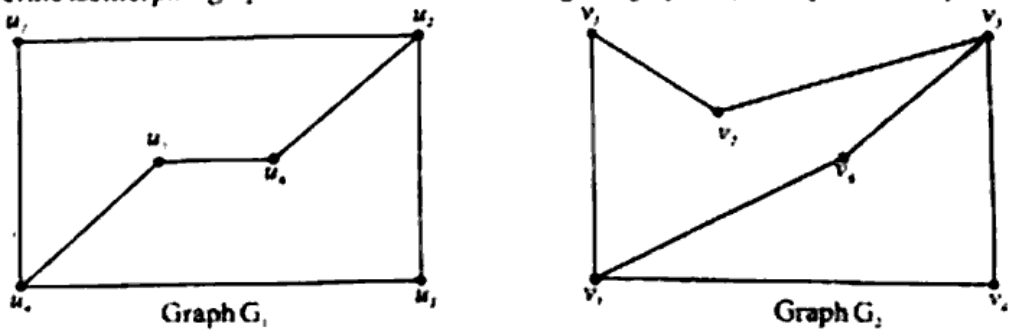
OR

(a) Find product $G_1 \times G_2$ and composition $G_1[G_2]$ of the following two graphs G_1 and G_2. Also write number of vertices and edges in the resulting graphs:



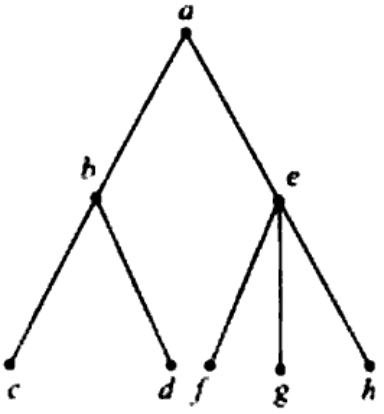
Graph G_1 Graph G_2

(b) Define isomorphic graphs. Show that the following two graphs G_1 and G_2 are isomorphic :

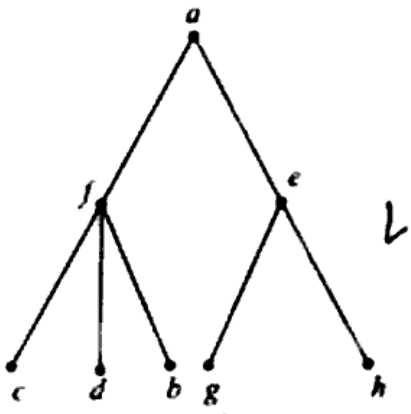


Graph G_1 Graph G_2

11. (a) What are the commonly used methods for tree traversal. Show that postorder traversals of the following two ordered rooted trees produce the same list of vertices :



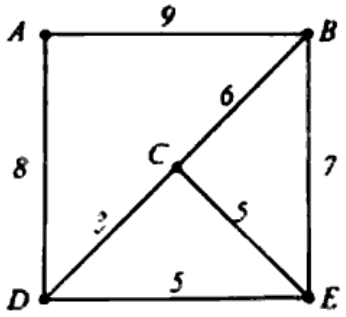
Tree 1



Tree 2

LRT

(b) Find the minimal spanning tree from the following graph by Prim's method :



OR

(a) Define the following with example:

(i) Leaf of a tree

(ii) Tree traversal

(iii) Path length of a binary tree

(b) What is the ordered rooted tree that represents the expression ((a+b) ↑ 2) + ((a-4) / 3). What is the value of the prefix expression + - * 2 3 5 / ↑ 2 3 4 ?